

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
NUMBER

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FURTHER MATHEMATICS

9231/02

Paper 2 Further Pure Mathematics 2

For examination from 2020

SPECIMEN PAPER

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **18** pages. Blank pages are indicated.

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1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2. \quad [6]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

2 Find the exact value of $\int_0^1 \frac{1}{\sqrt{3+4x-4x^2}} dx$.

[6]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

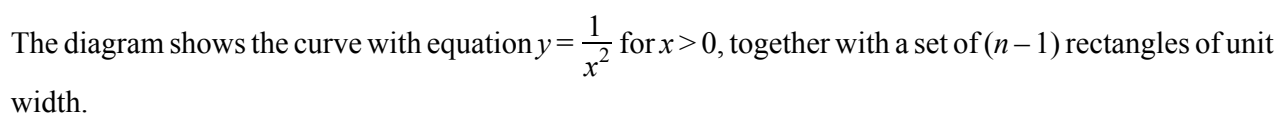
3 Find the solution of the differential equation

$$x \frac{dy}{dx} + 3y = \frac{\sin x}{x}$$

for which $y = 0$ when $x = \frac{1}{2}\pi$. Give your answer in the form $y = f(x)$.

[8]

[illegible]


$$\sum_{i=1}^n \frac{1}{r_i^2} < \frac{2n-1}{n}. \quad [5]$$
This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 The curve C has parametric equations

$$x = e^t - 4t + 3, \quad y = 8e^{\frac{1}{2}t}, \quad \text{for } 0 \leq t \leq 2.$$

(a) Find, in terms of e , the length of C .

[5]

[illegible]

- (b)** Find, in terms of π and e , the area of the surface generated when C is rotated through 2π radians about the x -axis. [5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

6 (a) Using de Moivre's theorem, show that

$$\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}. \quad [5]$$

[illegible]

[Turn over

- 7 (a) Starting from the definition of $\tanh$ in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]

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- (b) Given that $y = \tanh^{-1}\left(\frac{1-x}{2+x}\right)$, show that $(2x+1)\frac{dy}{dx} + 1 = 0$. [4]

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- [illegible]

where a , b and c are constants to be determined. [5]

- 8 (a) (i)** Find the set of values of a for which the system of equations

$$\begin{aligned}x - 2y - 2z + 7 &= 0, \\2x + (a - 9)y - 10z + 11 &= 0, \\3x - 6y + 2az + 29 &= 0,\end{aligned}$$

has a unique solution.

[4]

[illegible]

- (ii)** Given that $a = -3$, show that the system of equations in part **(i)** has no solution. Interpret this situation geometrically. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

- [4]

[illegible]

[illegible]

[illegible]

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